CHARACTERIZATION OF POTENTIAL SMOOTHNESS OF THE HILL OPERATOR AND RIESZ BASIS PROPERTY IN TERMS OF PERIODIC, ANTIPERIODIC AND NEUMANN SPECTRA

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Abstract: The Hill operators $Ly = -y'' + v(x)y$, considered with periodic, antiperiodic or Neumann boundary conditions have discrete spectra. For sufficiently large $n$ close to $n^2$ there are two periodic (if $n$ is even) or antiperiodic (if $n$ is odd) eigenvalues $\lambda_n^-$, $\lambda_n^+$, and respectively, one Neumann eigenvalue $\nu_n$. We study the geometry of "spectral triangle" with vertices ($\lambda_n^+, \lambda_n^-, \nu_n$), and show that the rate of decay of triangle size characterizes the potential smoothness. Moreover, it is proved that the periodic (antiperiodic) root function space contains a Riesz basis if and only if for even $n$ (respectively, for odd $n$)

$$\sup_{\lambda_n^+ \neq \lambda_n^-} \{|\lambda_n^+ - \nu_n|/|\lambda_n^+ - \lambda_n^-|\} < \infty.$$  

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