

İSTANBUL ANALYSIS SEMINARS

GAUSSIAN MEASURE vs LEBESGUE MEASURE AND ELEMENTS OF MALLIAVIN CALCULUS

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Abstract: Lebesgue measure does not make sense in infinite dimensions. However Gaussian measures do exist and have comparable properties. Besides they are natural, important in applications, therefore good substitutes for the Lebesgue measure. The main tools for generalizing a Gaussian measure from finite dimensions to Hilbert and Banach spaces are the Fourier transform (characteristic functional in the probability theory language) and the characterization of the covariance operators through nuclearity. The most important applications are the classical Wiener space and the abstract Wiener space.

To develop a differential calculus of Wiener functionals has some difficulties since many of the important functionals such as Itô integrals and the solutions of the stochastic differential equations are not Fréchet differentiable, may not even be continuous. Paul Malliavin initiated, by virtue of quasi-invariance of the Wiener measure (which is a particular Gaussian measure) a kind of weak differential calculus such that the foregoing functionals became smooth in a different sense. This was achieved by perturbing the Wiener functionals along the directions of vectors in a specific densely imbedded Hilbert subspace, called the Cameron-Martin space, thus taking the name of 'stochastic calculus of variations' or popularly known as Malliavin Calculus. Early successes included a nice probabilistic proof of Hörmander's theorem on hypoellipticity of differential operators, the existence and regularity of densities related to the laws of some Wiener functionals, smoothness of the heat kernel etc.

During the talk, basic terminology of the Malliavin Calculus such as the gradient, divergence and number operators, the Ornstein-Uhlenbeck semigroup, Meyer's inequalities, lifting of the tempered distributions in finite dimensions to infinite dimensional Watanabe generalized functions will be introduced to the extent that limited time permits.

Date: May 17, 2013

Time: 15:40

Place: Sabancı University, Karaköy Communication Center
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