Abstract: After the Russian mathematician I. Parovičenko, a compact Hausdorff space $X$ is called a Parovičenko space iff $X$ satisfies the following conditions:

1. $X$ is zero dimensional and has no isolated points,
2. $w(X) = 2^{\aleph_0}$,
3. every non-empty $G_\delta$ subset of $X$ has non-empty interior,
4. the closures of any two disjoint zero sets of $X$ are also disjoint.

The most well-known Parovičenko space is evidently the growth space $\mathbb{N}^* = \beta\mathbb{N} - \mathbb{N}$. In 1963 Parovičenko has proved the following astonishing fact: in the Set Theory Model $ZFC + (\aleph_1 = 2^{\aleph_0})$, all Parovičenko spaces are homeomorphic to $\mathbb{N}^*$ (see Soviet Math. Doklady 4 (1963), 592–595). Fifteen years later, E. van Douwen and J. van Mill, two young Dutch topologists, have proved the converse: i.e., if all Parovičenko spaces are homeomorphic (or equivalently, if there is no Parovičenko space other than $\mathbb{N}^*$), then the Continuum Hypothesis $\aleph_1 = 2^{\aleph_0}$ is true (see Proc. Amer. Math. Soc. 72 (1978), no. 3, 539–541). On the other hand, Canadian Murray Bell proved in 1990 that, there is a compact first countable Hausdorff space, which is not a continuous image of $\mathbb{N}^*$ in the model $ZFC + (\aleph_1 < 2^{\aleph_0})$ (see Topology Appl. 35 (1990), no. 2-3, 153–156).

We will give an explicit proof of van Douwen and van Mill’s outstanding result in this talk.

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