İSTANBUL ANALYSIS SEMINARS

PAROVIČENKO SPACES AND THE CONTINUUM HYPOTHESIS

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Abstract: After the Russian mathematician I. Parovičenko, a compact Hausdorff space X is called a ***Parovičenko space*** iff X satisfies the following conditions:

(1) X is zero dimensional and has no isolated points,

(2) $w(X) = 2^{\aleph_0}$,

(3) every non-empty G_δ subset of X has non-empty interior,

(4) the closures of any two disjoint zero sets of X are also disjoint.

The most well-known Parovičenko space is evidently the growth space $\mathbb{N}^* = \beta\mathbb{N} - \mathbb{N}$. In 1963 Parovičenko has proved the following astonishing fact: in the Set Theory Model ZFC$+(\aleph_1 = 2^{\aleph_0})$, all Parovičenko spaces are homeomorphic to $\mathbb{N}^*$ (see *Soviet Math. Doklady* **4** (1963), 592−595). Fifteen years later, E. van Douwen and J. van Mill, two young Dutch topologists, have proved the converse: i.e., if all Parovičenko spaces are homeomorphic (or equivalently, if there is no Parovičenko space other than $\mathbb{N}^*$), then the Continuum Hypothesis $\aleph_1 = 2^{\aleph_0}$ is true (see *Proc. Amer. Math. Soc.* **72** (1978), no. 3, 539−541). On the other hand, Canadian Murray Bell proved in 1990 that, there is a compact first countable Hausdorff space, which is *not* a continuous image of $\mathbb{N}^*$ in the model ZFC $+ (\aleph_1 < 2^{\aleph_0})$ (see *Topology Appl.* **35** (1990), no. 2-3, 153−156).

We will give an explicit proof of van Douwen and van Mill's outstanding result in this talk.

Date: March 6, 2015
Time: 15:40
Place: Sabancı University, Karaköy Communication Center
Bankalar Caddesi 2, Karaköy 34420, İstanbul

İstanbul Analysis Seminars is supported by TÜBİTAK.