Abstract: A Fourier quasicrystal is a pure point complex measure $\mu$ in $\mathbb{R}^p$ such that its Fourier transform in the sense of distributions $\hat{\mu}$ is also a pure point measure. For example, the sum $\mu$ of unit masses at the points of $\mathbb{Z}^p \subseteq \mathbb{R}^p$ is a Fourier quasicrystal, because $\hat{\mu}$ coincides with $\mu$ in this case.

There is a conjecture [1] that if supports of $\mu$ and $\hat{\mu}$ are both uniformly discrete sets, then the support of $\mu$ is a subset of a finite union of shifts of some full-rang lattice, and the support of $\hat{\mu}$ is a subset of a finite union of shifts of the conjugate lattice. This conjecture was proved in 2013 by N. Lev and A. Olevskii only for the cases of complex measures in $\mathbb{R}$ and for positive measures in $\mathbb{R}^p$, $p > 1$.

In my talk I will show that Lagarias’ conjecture is not valid in the general case and discuss some connected results.

References