

# İSTANBUL ANALYSIS SEMINARS

## GENERALIZED CONVEXITY, BLASCHKE-TYPE CONDITION IN UNBOUNDED DOMAINS, AND APPLICATION IN OPERATOR THEORY

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**Abstract:** It is known that zeros  $z_n$  of any bounded analytic function in the unit disc satisfy the Blaschke condition  $\sum(1 - |z_n|) < \infty$ . There are a lot of generalizations of Blaschke condition to unbounded analytic functions (M.M. Djrbashian, W. Hayman and B. Korenblum, F.A. Shamoyan, and many others).

We consider the case of analytic functions  $f$  in the unit disc growing near a subset  $E$  of the boundary and obtain an analog of the above condition as

$$\sum(1 - |z_n|)\rho^\kappa(z, E) < \infty,$$

where  $\rho(z, E)$  is the distance between  $z$  and  $E$ , and  $\kappa > 0$  depends only on the growth of  $f$  and the Minkowski dimension of the set  $E$ . Also, we show that our result is sharp.

Next, we introduce a notion of  $r$ -convexity for subsets of the complex plane. It is a pure geometric characteristic that generalizes the usual notion of convexity. Next, we investigate analytic and subharmonic functions that grow near the boundary in unbounded domains with  $r$ -convex compact complement. We obtain the Blaschke-type bounds for its Riesz measure and, in particular, for zeros of unbounded analytic functions in unbounded domains. These results are based on certain estimates for Green functions on complements of some neighborhoods of an  $r$ -convex compact set. Also, we apply our results in perturbation theory of linear operators in a Hilbert space. More precisely, we find quantitative estimates for the rate of condensation of the discrete spectrum of a perturbed operator near its essential spectrum.

This is joint work with L. Golinskii of Kharkov National University.

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