Let $E$ be a Banach function space on a probability measure space $(\Omega, \Sigma, \mu)$. Let $X$ be a Banach space and $E(X)$ be the associated Köthe-Bochner space. An operator on $E(X)$ is called a multiplication operator if it is given by multiplication by a function in $L^\infty(\mu)$. In the main result of this talk, we show that an operator $T$ on $E(X)$ is a multiplication operator if and only if $T$ commutes with $L^\infty(\mu)$ and leaves invariant the cyclic subspaces generated by the constant vector-valued functions in $E(X)$. As a corollary we show that this is equivalent to $T$ satisfying a functional equation considered by three Spanish mathematicians.

(joint work with Arkady Kitover and Mehmet Orhon)