IMBEDDING OPERATORS IN SOBOLEV-LIONS SPACES AND APPLICATIONS

The continuity and compactness of imbedding operators in Sobolev-Lions type spaces derived. The anisotropic weighted type spaces
\[ W^l_{p,\gamma}(\Omega; E_0, E) = W^l_{p,\gamma}(\Omega; E) \cap L^p_{\gamma}(\Omega; E_0), \]
is considered, where \( l = (l_1, l_2, \ldots, l_n) \), \( E_0 \) and \( E \) are two Banach spaces; \( E_0 \) is continuously and densely embedded into \( E \). Several conditions are found that ensure the continuity and compactness of embedding operators in these spaces in terms of interpolations of \( E_0 \) and \( E \). The most regular class of interpolation space \( E_\alpha \), between \( E_0 \) and \( E \) are found such that the mixed differential operators \( D^\alpha \) are bounded from \( W^l_{p,\gamma}(\Omega; E_0, E) \) to \( L^p_{\gamma}(\Omega; E_\alpha) \). These results are applied to anisotropic partial differential-operator equations and infinity systems of quasi elliptic equations with variable coefficients to obtain conditions that guarantee the maximal \( L^p_{\gamma} \) regularity.

Key Words: Banach-valued function spaces; Imbedding in Sobolev-Lions type spaces; Separable boundary value problems; Operators with discrete spectrum; Differential-operator equations; Operator-valued multipliers; Interpolation of Banach spaces; Semigroup of operators.