The Dirac operators

$$Ly = i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{dy}{dx} + v(x)y, \quad y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \quad x \in [0, \pi],$$

with L^2-potentials

$$v(x) = \begin{pmatrix} 0 & P(x) \\ Q(x) & 0 \end{pmatrix}, \quad P, Q \in L^2([0, \pi]),$$

considered on $[0, \pi]$ with periodic, antiperiodic or Dirichlet boundary conditions, have discrete spectrum, and the Riesz projections

$$P^* = \frac{1}{2\pi i} \int_{|z|=N-1/2} (z - L_{bc})^{-1} dz, \quad P_n = \frac{1}{2\pi i} \int_{|z-n|=1/4} (z - L_{bc})^{-1} dz$$

are well–defined for $|n| \geq N$ if N is sufficiently large. It is proved that

$$\sum_{|n|>N} \|P_n - P_n^0\|^2 < \infty,$$

where P_n^0, $n \in \mathbb{Z}$, are the Riesz projections of the free operator.

Then by the Bari–Markus criterion the spectral Riesz decompositions

$$f = P^* f + \sum_{|n|>N} P_n f, \quad \forall f \in L^2;$$

converge unconditionally in L^2.

The talk is based on recent joint results with B. Mityagin, see P. Djakov and B. Mityagin, Bari–Markus property for Riesz projections of 1D periodic Dirac operators, arXiv:0901.0856v1 [math.SP].