

Math 509
Hardy Spaces and Operator Theory
SPRING 2012/13

INSTRUCTOR:

Nihat Gökhan Göğüş

TIMETABLE:

Wednesday 8.40-11.30 (may change)

CONTACT:

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ASSESSMENT:

Participation

PRE-REQUISITES:

A graduate course in complex analysis (Rudin, Real and Complex Analysis) and preferably functional analysis (Conway, Functional Analysis)

PRINCIPAL TEXTBOOK:

P. L. Duren, Theory of H^p spaces, Pure and Applied Mathematics, Vol. 38 Academic Press, New York-London 1970

PROGRAMME

- Review of analytic functions. Review of analytic functions on the unit disk D , Blaschke products.
- Introduction to the Hardy space. Harmonic and subharmonic functions, Poisson integral, Littlewood's subordination theorem, the Hardy space $H_p(D)$ on D , the Nevanlinna class N .
 - Properties of the H_p spaces. Imbedding into $L_p(T)$, boundary values.
 - Factorization of H_p functions. Inner and outer functions, F. and M. Riesz theorem.
- Banach spaces. Banach spaces, Hilbert spaces, bounded linear operators, spectrum, invariant subspaces, the shift operator, Beurling's theorem.
- Singular inner functions. Structure of singular inner functions, closed invariant subspaces of H^2 .
 - Outer functions. Structure of outer functions.

The following are possible topics to continue. I am planning to choose according to the taste and interest of the course participants.

- H_p as a linear space. Representation of linear functionals and extremal problems.
 - Interpolation theory. Carleson's theorem.
- H_p spaces over general domains. Jordan and Smirnov domains, multiply connected domains.
 - The corona theorem. Maximal ideals.
- Composition operators. Basic properties of composition operators on H_p , compactness and invertibility of the composition operators.
- Spectra of composition operators. Eigenvalues and eigenvectors, spectra of composition operators.
 - More on composition operators. Topics will be determined in class.
- Toeplitz operators. Basic properties of Toeplitz operators, matrix representations, product of Toeplitz operators.
- Spectral structure. The spectral inclusion theorem, the Coburn alternative, spectra of self-adjoint Toeplitz operators and Toeplitz operators with continuous symbols.

SUGGESTED READING: 1) W. Rudin: Real and Complex Analysis, Mc-Graw Hill International Editions, 1987, ISBN: 0-07-100276-6.

2) R. A. Martinez-Avendano & P. Rosenthal: An Introduction to Operators on the Hardy-Hilbert Space, Springer Graduate texts in mathematics 237, 2000.

3) J. H. Shapiro: Composition Operators and Classical Function Theory, Springer-Verlag, Universitext: Tracts in mathematics, 1991, ISBN: 0-387-94067-7.

4) C. Cowen & B. MacCluer: Composition Operators on Spaces of Analytic Functions, CRC Press, Boca Raton, FL, 1995.