

İSTANBUL ANALYSIS SEMINARS

ASYMPTOTICS OF LEADING COEFFICIENTS OF ORTHONORMAL BASES IN SEVERAL COMPLEX VARIABLES

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Abstract: Let K be a pluriregular compact set in \mathbb{C}^n , $H \hookrightarrow A(K)$ a Hilbert space with dense embedding, and

$$p_i(z) = \sum_{j=1}^i a_{j,i} e_j(z), \quad i \in \mathbb{N}$$

be the orthonormal polynomial basis, obtained by Gram-Schmidt procedure from the system of monomials $e_i(z) := z^{k(i)} = z_1^{k_1(i)} \dots z_n^{k_n(i)}$ enumerated so that the sequence of degrees $s(i) := k_1(i) + \dots + k_n(i)$ is non-decreasing and the monomials of the same degree are ordered lexicographically. Under some natural and quite general assumptions about H the system $\{p_i\}$ is a basis in the space $A(K)$ and in the spaces $A(D_R)$, where D_R , $R > 1$, are sublevel domains of the pluripotential Green function $g_K(z)$ ([Zakharyuta 75, Zeriahi 85]). We discuss the results about asymptotics of the leading coefficients a_{ii} in terms of *Chebyshev directional constants and Fekete-Leja transfinite diameter* ([Zakharyuta 76, Zeriahi 85, Zakharyuta 94, Zakharyuta 12]).

We discuss also the recent results ([Zakharyuta 12a]) on asymptotics of leading coefficients of orthonormal bases in the space $A(D)$ for a domain $D \subset \mathbb{C}^n$, which have the following expansion

$$\varphi_i(z) = \sum_{j=i}^{\infty} a_{j,i} e_j(z - a)$$

near a given point $a \in D$. These asymptotics are given in terms of *interior Chebyshev directional constants and interior transfinite diameter of a domain D relative to $a \in D$* .

References

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