

İSTANBUL ANALYSIS SEMINARS

TRANSFINITE DIAMETER, CHEBYSHEV CONSTANTS AND CAPACITIES IN \mathbb{C}^n

Vyacheslav P. ZAKHARYUTA

Sabancı University
Faculty of Engineering and Natural Sciences

Abstract: One of the fundamental results in complex analysis is the classical result of the first third of 20th century (Fekete, Szegő et al) which says that *for any compact set $K \subset \mathbb{C}$ the transfinite diameter $d(K)$, the Chebyshev constant $\tau(K)$ and the capacity $c(K)$ coincide*, although they are defined from very different points of view. Indeed, the transfinite diameter derives from a geometrical approach as the *limit of geometrical means of some extremal distances between points of K* :

$$\begin{aligned} d(K) &:= \lim_{s \rightarrow \infty} d_s(K), \\ d_s(K) &:= \left(\max \left\{ \left| \det \left(z_j^{k-1} \right)_{k,j=1}^s \right| : z_j \in K \right\} \right)^{2/s(s-1)}; \end{aligned} \quad (1)$$

the Chebyshev constant is defined in *terms of the least uniform deviation of monic polynomials from zero*:

$$\tau(K) := \lim_{s \rightarrow \infty} \inf \left\{ \max_{z \in K} \left| z^s + \sum_{j < s} c_j z^j \right| : c_j \in \mathbb{C}, j = 1, \dots, s-1 \right\}^{1/s},$$

while the capacity appears from the *potential theory* considerations:

$$c(K) := \exp(-\lambda(K)), \quad \lambda(K) := \lim_{z \rightarrow \infty} (g_K(z) - \ln|z|)$$

where $g_K(z)$ is the Green function for K with a logarithmic singularity at ∞ .

For a compact set K in \mathbb{C}^n , the transfinite diameter was introduced by F. Leja in 1957: $d(K) := \limsup_{s \rightarrow \infty} d_s(K)$, where $d_s(K)$ is determined analogously to (1). He posed a problem whether there exists the usual limit in his definition. This problem has been solved positively by the speaker in 1975, it was shown also that the *Leja transfinite diameter* coincides with, so-called, *principal Chebyshev constant*, which is expressed as a continual geometric mean of *directional Chebyshev constants*.

In my talk I give a survey of results concerned with the above notions for several complex variables (Bloom, Bos, Levenberg, Calvi, Zeriahi, Rumely, Lau, Varley, Berman, Boucksom, Nystrom et al) and discuss a new approach to the notions of the transfinite diameter and Chebyshev constants on Stein manifolds.

Date: April 1, 2011

Time: 15:40

Place: Sabancı University, Karaköy Communication Center
Bankalar Caddesi 2, Karaköy 34420, İstanbul