## Some Results in Metric Trees

The study of injective envelopes of metric spaces, also known as metric trees (R-trees or T-theory), has its motivation in many subdisciplines of mathematics as well as biology/medicine and computer science. Its relationship with biology and medicine stems from the construction of phylogenetic trees [5].Concepts of "string matching" in computer science is closely related with the structure of metric trees [4]. A metric tree is a metric space $(M, d)$ such that for every $x, y$ in $M$ there is a unique arc between $x$ and $y$ and this arc is isometric to an interval in $\mathbb{R}$ [3],[2]. In this talk, we examine convexity and compact structures in metric trees and show that nonempty closed convex subsets of a metric tree enjoy many properties shared by convex subsets of Hilbert spaces and admissible subsets of hyperconvex spaces. We show that a set valued mapping $T^*$ of a metric tree $M$ with convex values has a selection $T : M \to M$ for which $d(T(x), T(y)) \leq d_H(T^*(x), T^*(y))$ for each $x, y \in M$. Here by $d_H$ we mean the Hausdroff distance [1]. We will mention some applications to k-set contractions as well as an application of the above selection theorem. Furthermore we define n-widths $\delta_n(A)$ of a subset $A$ of a metric tree $M$ and show that even in the absence of linear structure the limit of n-widths as $n \to \infty$ is equal to the ball measure of noncompactness.

### References


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