Some Results in Metric Trees

The study of injective envelopes of metric spaces, also known as metric trees (R-trees or T-theory), has its motivation in many subdisciplines of mathematics as well as biology/medicine and computer science. Its relationship with biology and medicine stems from the construction of phylogenetic trees [5]. Concepts of “string matching” in computer science is closely related with the structure of metric trees [4]. A metric tree is a metric space \((M,d)\) such that for every \(x, y \in M\) there is a unique arc between \(x\) and \(y\) and this arc is isometric to an interval in \(\mathbb{R}\) [3],[2]. In this talk, we examine convexity and compact structures in metric trees and show that nonempty closed convex subsets of a metric tree enjoy many properties shared by convex subsets of Hilbert spaces and admissible subsets of hyperconvex spaces. We show that a set valued mapping \(T^*\) of a metric tree \(M\) with convex values has a selection \(T: M \to M\) for which \(d(T(x),T(y)) \leq d_H(T^*(x),T^*(y))\) for each \(x,y \in M\). Here by \(d_H\) we mean the Hausdorff distance [1]. We will mention some applications to k-set contractions as well as an application of the above selection theorem. Furthermore we define n-widths \(\delta_n(A)\) of a subset \(A\) of a metric tree \(M\) and show that even in the absence of linear structure the limit of n-widths as \(n \to \infty\) is equal to the ball measure of noncompactness.

References


