

# Approximation Properties and Bounded Holomorphic Mappings in Banach Spaces

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**Abstract.** Let  $E$  and  $F$  be Banach spaces and let  $\tau_c$  denote the compact-open topology on the vector space of all linear continuous operators  $L(E; F)$ . Then we have the following characterizations of the (compact) approximation property:

(i) (Linear case)  $E$  has the (compact) approximation property if and only if, for every Banach space  $F$ , every linear continuous operator in  $L(E; F)$  lies in the  $\tau_c$ -closure of the subspace of all finite rank (resp. compact) linear operators  $E' \otimes F$  (resp.  $L_k(E; F)$ ).

(ii) (Holomorphic case)  $E$  has the (compact) approximation property if and only if each continuous Banach-valued polynomial on  $E$  can be uniformly approximated on compact sets by finite type (resp. compact) polynomials if and only if, for every bounded balanced open subset  $U$  of  $E$ , the predual of the space of the bounded holomorphic mappings on  $U$ ,  $G^\infty(U)$ , has the (compact) approximation property.

Analogous results hold for the bounded (compact) approximation property also.