Kolmogorov problem on widths asymptotics

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Given a compact set K in an open set D on a Stein manifold $\Omega, \dim \Omega = n$, the set A_K^D of all restrictions of functions, analytic in D with absolute value bounded by 1, is considered as a compact subset in $C(K)$. The problem about the strict asymptotics for Kolmogorov diameters:

$$-\ln d_s\left(A_K^D\right) \sim \sigma\ s^{1/n},\ s \to \infty. \tag{1}$$

was stated by Kolmogorov (in an equivalent formulation for ε-entropy of this set). It was conjectured in [4,5] that for "good" pairs $K \subset D$ the asymptotics (1) holds with $\sigma = 2\pi\left(\frac{n!}{C(K,D)}\right)^{1/n}$, where $C(K,D)$ is the pluricapacity of the "pluricondenser" (K,D) (see, e.g., [1]). In the one-dimensional case this hypothesis is equivalent to Kolmogorov's conjecture about asymptotics of ε-entropy of the set A_K^D, which has been confirmed by efforts of many authors (Erokhin, Babenko, Zaharyuta, Levin-Tikhomirov, Widom, Nguyen, Skiba - Zahariuta, Fisher - Miccheli, et al).

In [4,5] the problem (1) was reduced (the proof was only sketched, for a detailed proof see, [6]) to the problem of Pluripotential Theory about approximation of the relative Green pluripotential of the pluricondenser (K,D) by pluripotentials with finite set of logarithmic singularities. The last problem has been solved recently by Nivoche-Poletsky ([2,3]), which results, together with [4,5,6], a final proof of our conjecture about strong asymptotics (1) under some natural restrictions about K, D.

In our talk attention is focused on a proper modification of the Kolmogorov problem, which is connected more tightly with certain natural pluripotential properties of a pluricondenser (K,D); the asymptotics (1) is treated in the frame of those more general considerations. On the other hand, considerable attention is given to the important device in the proof of the reduction - special extendible bases with good estimates on sublevel domains of a multipolar pluricomplex Green function.

References

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