DYNAMIC RAYS AND LANDING BEHAVIORS

Aslı Deniz

February 20, 2015

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Escaping set:

$$\mathcal{I}(f) := \{z; \quad f^n(z) \rightarrow \infty\}.$$

Outline

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- Rays for Polynomials

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- Rays for Transcendental Entire Functions

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    - * A Landing Theorem

RAYS FOR POLYNOMIALS

Quadratic family

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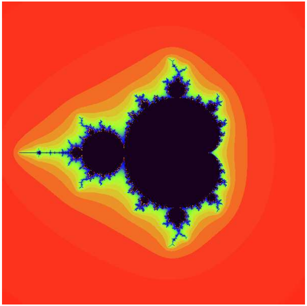

Mandelbrot Set

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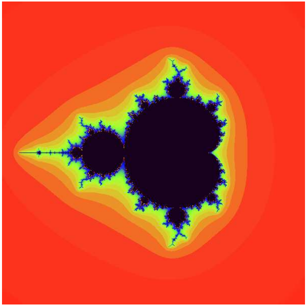

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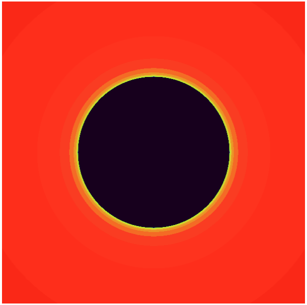

Dynamical plane for Q_0

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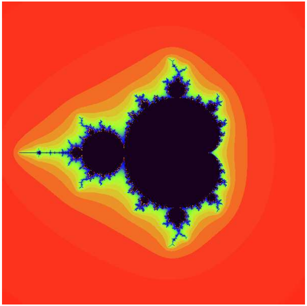

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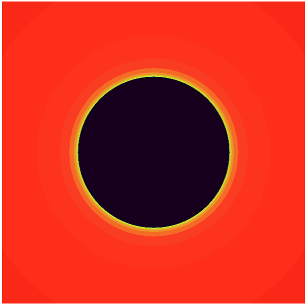

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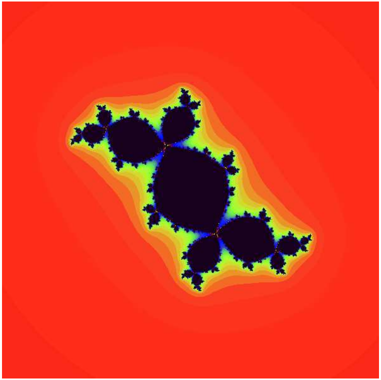

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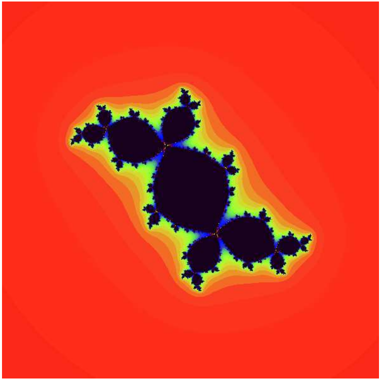

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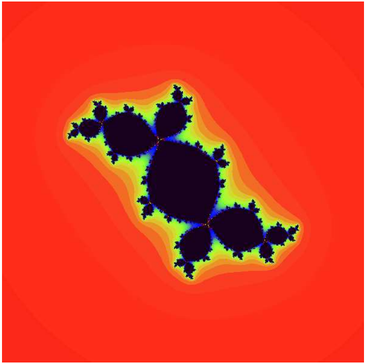
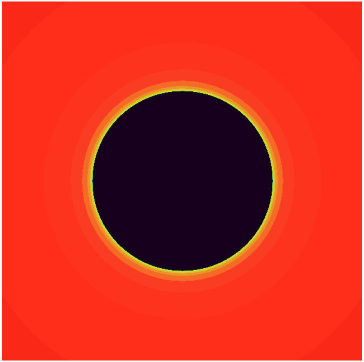
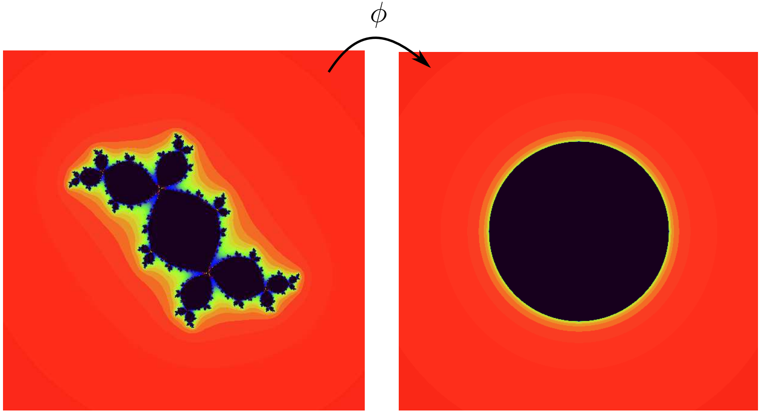
Aim: To understand the topology of the Julia set..





In the escaping sets the dynamics are "similar"...

Exploring the Julia set



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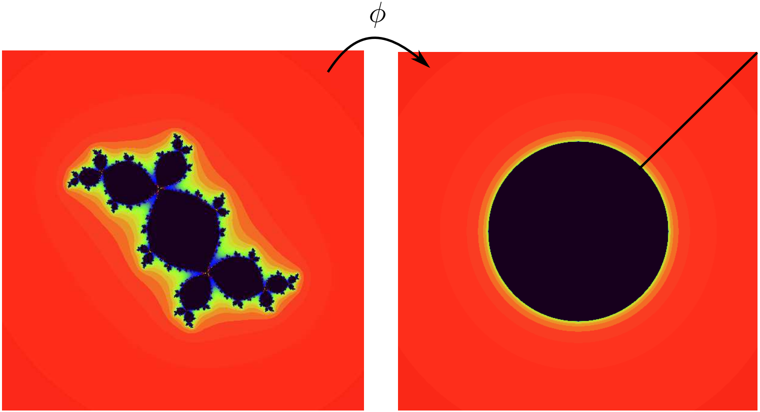

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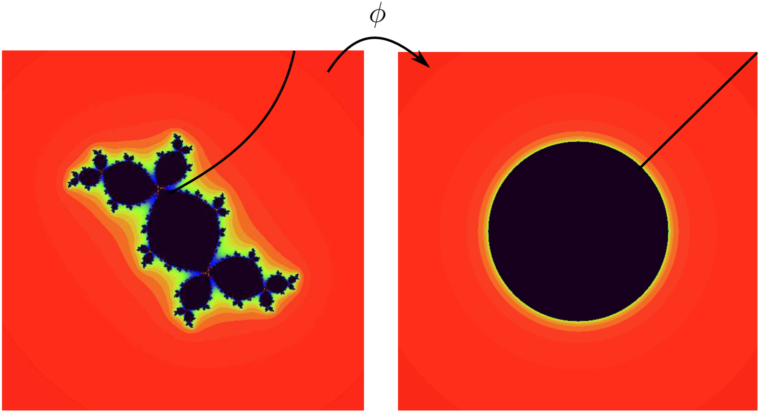

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Dynamic rays with period 3

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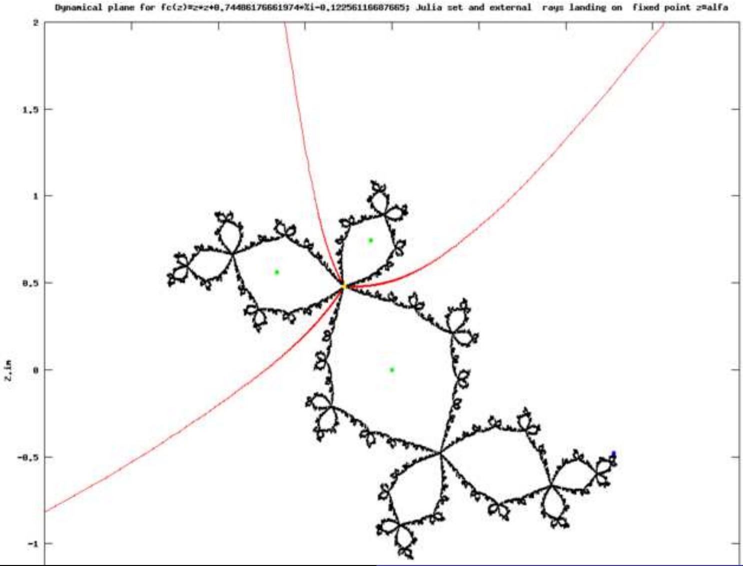

DYNAMIC RAYS ARE WAYS TO REACH THE JULIA SET...

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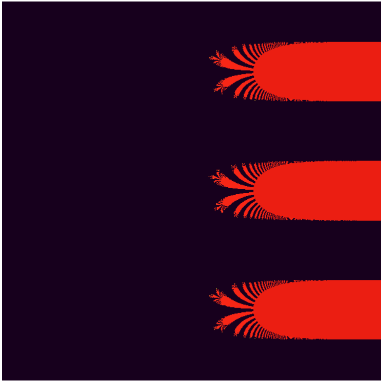
Theorem (Sullivan-Douady-Hubbard)

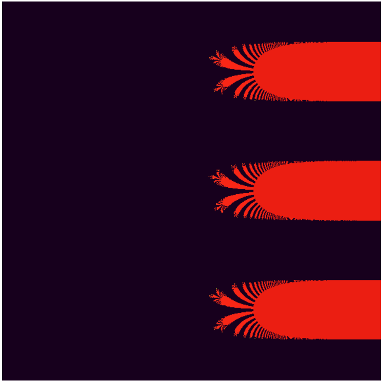
If the critical value has bounded orbit, then every periodic ray lands at a periodic point.

RAYS FOR TRANSCENDENTAL ENTIRE FUNCTIONS

Transcendental Dynamics



Exponential dynamics



Exponential dynamics

Theorem (Eremenko-Lyubich)

Suppose f is a transcendental entire function with bounded singular set. Then

$$\mathcal{J}(f) = \overline{\mathcal{I}(f)}.$$

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Call it R^3S class...

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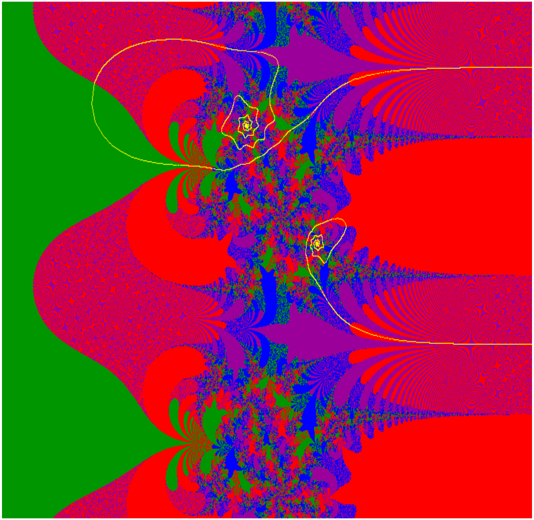

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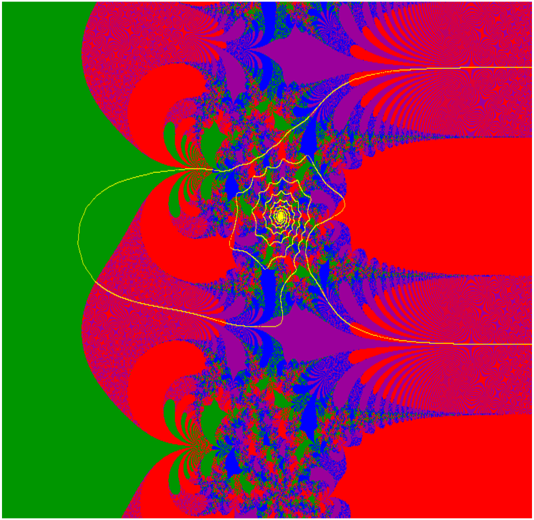

Exponential dynamics

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Set post-singular set:

$$P = \overline{\bigcup_n f^n(S)}.$$

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Answer: a single point...

THANK YOU FOR YOUR ATTENTION!