DYNAMIC RAYS AND LANDING BEHAVIORS

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What is holomorphic dynamics?
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- the set of points with stable behavior $\rightarrow$ **Fatou set** ($\mathcal{F}(f)$)
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   \[
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- the set of points with stable behavior \( \rightarrow \text{Fatou set} \ (\mathcal{F}(f)) \)
- the set of points with non-stable behavior \( \rightarrow \text{Julia set} \ (\mathcal{J}(f)) \)
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**Escaping set:**

$\mathcal{I}(f) := \{ z; \ f^n(z) \rightarrow \infty \}.$
Rays for Polynomials
Outline

- Rays for Polynomials
- Rays for Transcendental Entire Functions
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- Rays for Transcendental Entire Functions
  - A Landing Theorem
RAYS FOR POLYNOMIALS
Quadratic family
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\[ Q_c(z) = z^2 + c, \quad c \in \mathbb{C}. \]
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Mandelbrot Set

\[ \mathcal{J}(Q_0) = \partial \mathcal{I}(Q_0) \]

Dynamical plane for \( Q_0 \)
$$\mathcal{J}(Q_c) = \partial \mathcal{I}(Q_c)$$
Quadratic family

\[ J(Qc) = \partial I(Qc) \]

Douady’s rabbit
Quadratic family

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Douady’s rabbit

Aim: To understand the topology of the Julia set..
Exploring the Julia set

In the escaping sets the dynamics are "similar"...
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Dynamic rays with period 3
Exploring the Julia set

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DYNAMIC RAYS ARE WAYS TO REACH THE JULIA SET...
Exploring the Julia set

Dynamic rays are ways to reach the Julia set...

Theorem (Sullivan-Douady-Hubbard)

If the critical value has bounded orbit, then every periodic ray lands at a periodic point.
RAYS FOR TRANSCENDENTAL ENTIRE FUNCTIONS
Exponential dynamics
Theorem (Eremenko-Lyubich)

Suppose $f$ is a transcendental entire function with bounded singular set. Then

$$\mathcal{J}(f) = \mathcal{I}(f).$$

Exponential dynamics
QUESTION: DO ESCAPING POINTS FORM CURVES?
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ANSWER: For some classes of functions..YES!
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Theorem (Rottenfusser, Rückert, Rempe, Schleicher)

Dynamic rays exist for functions of finite order with bounded singular sets, or finite composition of such functions.
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Call it $R^3S$ class...
Exponential dynamics
Transcendental Dynamics

Exponential dynamics
Landing theorem via hyperbolic geometry
Set post-singular set:

\[ P = \bigcup_{n} f^n(S). \]
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Theorem (D.)

For \( f \in R^3S \) with bounded post-singular set, every periodic dynamic ray lands at a periodic point.
Landing theorem via hyperbolic geometry

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**Theorem (D.)**

*For \( f \in R^3 S \) with bounded post-singular set, every periodic dynamic ray lands at a periodic point.*

- **Tools:** Standard hyperbolic geometry results
Landing theorem via hyperbolic geometry

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Tools: Standard hyperbolic geometry results
- Schwarz-Pick’s Lemma
- Comparison Principle
Landing theorem via hyperbolic geometry

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- **Tools:** Standard hyperbolic geometry results
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- **Strategy:** Perform successive pullbacks of a fundamental segment and see in the limit it degenerates to a single point.
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Landing theorem via hyperbolic geometry

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**Answer:** a single point...
THANK YOU FOR YOUR ATTENTION!