THE SPECTRAL NEVANLINNA–PICK PROBLEM, THE SPECTRAL BALL AND THE SYMMETRIZED POLYDISC

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Let Ω_n be the set of all $n \times n$ complex matrices with spectral radius less than 1. The spectral Nevanlinna–Pick problem is the following one: given N points $a_1, \dots, a_N$ in the unit disk $\mathbb{D} \subset \mathbb{C}$ and N matrices $A_1, \dots, A_N \in \Omega_n$ decide whether there is a holomorphic mapping $\varphi : D \to \Omega_n$ such that $\varphi(a_1) = A_1, \dots, \varphi(a_1) = A_N$.

The study of the Nevanlinna–Pick problem in the case $N = 2$ reduces to the computation of the so-called Lempert function l_{Ω_n} of Ω_n.

In this talk we shall give an explicit formula for l_{Ω_2}. It will be done by computing the Lempert function $l_{\mathbb{G}_2}$ of the so-called symmetrized bidisc $\mathbb{G}_2$ - the image of the bidisc under the mapping with components the two elementary symmetric functions of two complex variables. This domain serves as the first counterexample to the converse of the famous Lempert theorem - $l_{\mathbb{G}_2}$ coincides with the Carathéodory distance of $\mathbb{G}_2$ but $\mathbb{G}_2$ cannot be exhausted by domains biholomorphic to convex domains.

We shall also discuss the above problems for $n > 2$.