RIESZ BASES CONSISTING OF ROOT FUNCTIONS OF 1D DIRAC OPERATORS WITH REGULAR BOUNDARY CONDITIONS

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This talk is based on recent results obtained in collaboration with Boris Mityagin (Ohio State University, USA) – see [1].

We study Dirac operators

$$Ly = i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{dy}{dx} + v(x)y, \quad y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \quad x \in [0, \pi],$$

with L^2-potentials

$$v(x) = \begin{pmatrix} 0 & P(x) \\ Q(x) & 0 \end{pmatrix}, \quad P, Q \in L^2([0,\pi]),$$

considered on $[0, \pi]$ with regular boundary conditions. They have discrete spectrum, and the Riesz projections

$$P_n = \frac{1}{2\pi i} \int_{C_n} (z - L)^{-1} dz, \quad C_n = \{z : \ |z - n| = 1/2\}$$

are well defined for $|n| \geq N$ if N is sufficiently large. Our main result says that the Bari–Markus property holds, i.e.,

$$\sum_{|n|>N} \|P_n - P_n^0\|^2 < \infty,$$

where P_n^0, $n \in \mathbb{Z}$, are the Riesz projections of the free operator. From here it follows, for strictly regular boundary conditions, that the system of normalized root functions is a Riesz basis.

[1] Plamen Djakov and Boris Mityagin, Unconditional Convergence of Spectral Decompositions of 1D Dirac Operators with Regular Boundary Conditions, Oberwolfach Preprints 2010 – 21, 38 pages.