İSTANBUL ANALYSIS SEMINARS

ON HADAMARD TYPE MULTIPLIERS FOR REAL ANALYTIC FUNCTIONS OF SEVERAL VARIABLES

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Abstract: Any linear (continuous) map $M : A(\Omega) \to A(\Omega)$ such that all monomials $x^\alpha := x_1^{\alpha_1} \dots x_d^{\alpha_d}$ are eigenvectors is called a (Hadamard type) multiplier on the space $A(\Omega)$ of real analytic functions over an open set $\Omega \subset \mathbb{R}^d$. By (m_α) we denote the corresponding multisequence of eigenvalues. If $(0, \dots, 0) \in \Omega$ then for each real analytic f holds $M(f)(z_1, \dots, z_d) = \sum_\alpha m_\alpha a_\alpha z^\alpha$ around zero whenever $f(z_1, \dots, z_d) = \sum_\alpha a_\alpha z^\alpha$ around zero. Since monomials do not form a Schauder basis in $A(\Omega)$ it is by far not clear which sequences (m_α) correspond to some well defined M although the sequence of eigenvalues (m_α) uniquely identifies M.

We will survey results on Hadamard type multipliers starting with several examples. In particular, we show that multipliers M 1-1 correspond via an explicit formula to analytic functionals T (i.e., linear continuous functionals on a suitable space of real analytic functions) so that the sequence of eigenvalues (m_α) corresponds to the sequence of moments of the analytic functional $(\langle T, x^\alpha \rangle)_\alpha$ or to Laurent coefficients of some functions holomorphic at infinity. We explain to which analytic functionals correspond Euler differential operators (a type of linear differential operators with variable coefficients). Then we characterize sequences (m_α) of moments of analytic functionals in terms of holomorphic functions interpolating values m_α at points in the positive integer lattice $\mathbb{N}^d$ −the latter result can be also treated as an analogue of a classical result of Carlson on interpolation of Taylor coefficients of some holomorphic functions of one variable. This characterization allows us to get sufficient and necessary conditions on multipliers to be surjective. In consequence, we get several examples of linear partial differential operators of variable coefficients (Euler type operators) which are surjective or which are not surjective on the space $A(\mathbb{R}^d)$.

The research is a part of a joint project with M. Langenbruch, some results are obtained jointly also with D. Vogt.

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