İSTANBUL ANALYSIS SEMINARS

UNIFORMLY SPREAD DISCRETE SETS

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Abstract: We consider discrete infinite sets in $\mathbb{R}^p$, i.e., infinite sets without finite limit points. Such sets appear in various branches of analysis (zero and pole sets of meromorphic functions, various models in the mathematical theory of quasicrystals, and so on). For arbitrary infinite discrete sets $A = (a_n)_{n\in I}$ and $B = (b_n)_{n\in I}$ put $d(A,B) = \inf \sup_n |a_n - b_{\sigma(n)}|$, where inf is taken over all bijections σ of the index set I. Actually, the condition $d(A,B) < \infty$ means that B is a bounded perturbation of the set A.

Theorem. *If the distances $d(A, A+t)$ are bounded uniformly in $t \in \mathbb{R}^p$, then*

(a) *a finite positive density $\Delta = \lim_{T\to\infty} \operatorname{card}\{n : |a_n - c| < T\}(\omega_p T^p)^{-1}$ exists uniformly with the respect to $c \in \mathbb{R}^p$ (ω_p is the volume of the unit ball in $\mathbb{R}^p$);*

(b) *A is a uniformly spread set in the sense of Laczkovich, i.e., $d(A, \Delta^{-1/p}\,\mathbb{Z}^p) < \infty$.*

Theorem. *If a discrete set $A \subset \mathbb{R}^p$ is almost periodic (i.e., for any continuous function φ with a compact support the sum $\sum_n \varphi(x+a_n)$ is almost periodic in $x \in \mathbb{R}^p$) and the set of differences $A-A$ is discrete, then $A = L+F$, where F is finite, and L is a full-rank lattice in $\mathbb{R}^p$.*

Theorem. *If differences of zeros of an exponential sum*

$$\sum_{n=1}^{N} a_n e^{i\lambda_n z}, \quad \lambda_n \in \mathbb{R}, \quad z \in \mathbb{C},$$

form a discrete set, then the sum has the form

$$Ke^{i\beta z}\prod_{k=1}^{N} \sin(\omega z + b_k), \quad \omega, \beta \in \mathbb{R}, \quad K, b_k \in \mathbb{C}.$$

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