

İSTANBUL ANALYSIS SEMINARS

ASPECTS OF OPERATOR THEORY ON WEIGHTED SYMMETRIC FOCK SPACES

H. Turgay KAPTANOĞLU

Bilkent University
Department of Mathematics

Abstract: The weighted symmetric Fock spaces \mathcal{F}_b considered with weight sequence $b = \{b_k\}$ are realized as reproducing kernel Hilbert spaces of holomorphic functions on the unit ball \mathbb{B} of \mathbb{C}^N . Some of them are the Dirichlet spaces \mathcal{D}_q with kernels

$$K_q(z, w) = (1 - \langle z, w \rangle)^{-(1+N+q)}$$

for $q > -(1 + N)$ and hypergeometric functions for $q \leq -(1 + N)$, and include the Drury-Arveson space, Dirichlet space, and Hardy and Bergman Hilbert spaces. We use more function theory than usual to do operator theory on \mathcal{F}_b . Radial differential operators are prominent throughout.

Our results concern the N -tuple of operators of multiplication by the coordinate functions on each weighted symmetric Fock space \mathcal{F}_b called the shift S_b . We characterize the commutant of the shift S_b as the multiplier algebra of \mathcal{F}_b . We determine the analytic Hilbert modules among the \mathcal{F}_b . A von Neumann inequality is an upper bound on the norm of a polynomial of a row contraction of operators on an arbitrary Hilbert space. We obtain a von Neumann inequality in terms of the multiplier norm of a polynomial of S_b for any b .

Specializing to Dirichlet spaces, we find the \mathcal{D}_q on which the shift S_q is subnormal. We show that the C^* -algebra \mathcal{T}_q of operators on \mathcal{D}_q generated by S_q contains all compact operators and certain generalized Toeplitz operators with continuous symbols. We obtain the related short exact sequence of C^* -algebras for any $q \in \mathbb{R}$ and show that they all lie in the same Ext class.

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Bankalar Caddesi 2, Karaköy 34420, İstanbul