The Dixmier Trace

Mohan Ravichandran, Sabanci University

Jacques Dixmier in 1966, construced traces on $\mathcal{B}(\mathcal{H})$, where $\mathcal{H}$ is infinite dimensional that are not proportional to the usual trace(Any normal trace on $\mathcal{B}(\mathcal{H})$ is proportional to the usual trace). These traces are defined on an ideal of the compact operators, denoted $\mathcal{L}_1^+$, vanish on the trace class operators and measure the logarithamic divergence of the $s-$numbers of an operator. While Dixmier traces depend upon the choice of a Banach limit, for certain naturally occurring classes of operators, the number is independent of the choice of limit.

These traces were popularised by Alain Connes, who used these to develop a non-commutative infinitesimal calculus. Further, Connes showed that the residue of pseudodifferential operators (acting on sections of complex vector bundles) on manifolds, introduced by Manin, Wodzicki and Guillemin equals the Dixmier trace.

This talk will be a gentle introduction to the Dixmier trace, its properties and some of its applications.