CONVERGENCE OF SPECTRAL DECOMPOSITIONS OF HILL OPERATORS WITH TRIGONOMETRIC POLYNOMIAL POTENTIALS

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We consider the Hill operator

$$Ly = -y'' + v(x)y, \quad 0 \le x \le \pi,$$

subject to periodic or antiperiodic boundary conditions, with potentials v which are trigonometric polynomials with nonzero coefficients, of the form

(i) $ae^{-2ix} + be^{2ix}$; (ii) $ae^{-2ix} + Be^{4ix}$; (iii) $ae^{-2ix} + Ae^{-4ix} + be^{2ix} + Be^{4ix}$.

Then the system of eigenfunctions and (at most finitely many) associated functions is complete but it is not a basis in $L^2([0,\pi],\mathbb{C})$ if $|a| \neq |b|$ in the case (i), if $|A| \neq |B|$ and neither $-b^2/4B$ nor $-a^2/4A$ is an integer square in the case (iii), and it is never a basis in the case (ii) subject to periodic boundary conditions.

This talk presents recent results obtained in a collaboration with Boris Mityagin (Ohio State University, USA).