İSTANBUL ANALYSIS SEMINARS

ON THE REFLEXIVITY, HYPERREFLEXIVITY AND TRANSITIVITY OF TOEPLITZ OPERATORS

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Abstract: Let H be a complex Hilbert space and let $L(H)$ denote the algebra of all bounded linear operators on H. Let us consider $\mathcal{W}$ a unital subalgebra of $L(H)$ closed in WOT topology. Denote by $\operatorname{Lat}\mathcal{W}$ the set of all closed subspaces invariant for operators from $\mathcal{W}$, and by $\operatorname{Alg}\operatorname{Lat}\mathcal{W}$ denote the algebra containing all operators $T \in L(H)$ leaving invariant all subspaces from $\operatorname{Lat}\mathcal{W}$. We have always $\mathcal{W} \subset \operatorname{Alg}\operatorname{Lat}\mathcal{W} \subset L(H)$. An algebra, following D. Sarason, is called *reflexive* if $\mathcal{W} = \operatorname{Alg}\operatorname{Lat}\mathcal{W}$. In other words an algebra $\mathcal{W}$ is reflexive if it has so many invariant subspaces that they determine the algebra itself. On the contrary an algebra $\mathcal{W}$ is called transitive if $\operatorname{Lat}\mathcal{W} = \{H, \{0\}\}$, i.e., $\operatorname{Alg}\operatorname{Lat}\mathcal{W} = L(H)$. Now for a given operator $A \in L(H)$ except the usual distance from A to $\mathcal{W}$ denoted by $\operatorname{dist}(A, \mathcal{W})$, we can define the distance "determined by its invariant subspaces" as $\alpha(A, \mathcal{W}) = \sup\{\|(I-P)AP\| : P \in \operatorname{Lat}\mathcal{W}\}$. Usually $\alpha(A, \mathcal{W}) \leqslant \operatorname{dist}(A, \mathcal{W})$. W. Arveson called an algebra *hyperreflexive* if the usual distance can be controlled by the distance α. The hyperreflexivity is a stronger notion than reflexivity.

If fact there is no need to have the algebra structure for reflexivity, hyperreflexivity and transitivity. More suitable setting for this notions are not algebras, but subspaces of operators. It was observed by V. Schulman and D. Larson. Moreover, there are weaker, having the same origin properties: k-reflexivity and k-hyperreflexivity.

The recent reflexivity, transitivity and hyperreflexivity results for subspaces and algebras of Toeplitz operators will be presented. We start with the classical result about reflexivity and hyperreflexivity of analytic Toeplitz operators on the Hardy space on the unit disc. The space of all Toeplitz operators is transitive but 2-reflexive. We will study the dichotomic behavior of subspaces of Toeplitz operators on the Hardy space. A linear space of Toeplitz operators which is closed in the ultraweak operator topology is either transitive or reflexive. No intermediate behavior is possible. This result can be extended to the Toeplitz operators on the Hardy space on the upper half-plane. The Toeplitz operators on the Bergman space will be also considered. The generalized Toeplitz and the multivariable Toeplitz operators case will be also considered.

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