Kadison-Singer algebras: A Generalization of triangular algebras

Mohan Ravichandran, Sabanci University

Kadison and Singer introduced the class of Triangular algebras in 1960, intitiating the systematic study of non-selfadjoint operator algebras. The prototype of a triangular algebra is the algebra of bounded operators, upper triangular with respect to a given orthonormal basis in a Hilbert space. They called a subalgebra $\mathfrak{B}$ of $\mathcal{B}(\mathcal{H})$ triangular if $\mathfrak{B} \cap (\mathfrak{B})^*$ is a maximal abelian self-adjoint subalgebra(masa in short) of $\mathcal{B}(\mathcal{H})$. I will begin the talk with a a survey of the theory of triangular algebras.

In 2008, Ge and Yuan, seeking a more intimate connection between non-selfadjoint and self-adjoint algebras, introduced the class of Kadison-Singer algebras. Given a von Neumann algebra $\mathfrak{M} \subseteq B(H)$, a Kadison-Singer algebra $\mathfrak{A}$ is a maximal reflexive algebra with diagonal $\mathfrak{M}$, ie, $\mathfrak{A}^* \cap \mathfrak{A} = \mathfrak{M}$. Kadison-Singer algebras generalize the most interesting class of triangular algebras and also generalize the class of nest algebras, which have both been extensively studied. I will show how the study of these algebras throws up some tantalizing connections between the theories of non-selfadjoint and von Neumann algebras.

I will construct several Kadison-Singer algebras, including ones with diagonal the hyperfinite(both type II_1 and the Powers factors) and free group factors. The lattice of invariant projections of a Kadison-Singer algebra is called a Kadison-Singer lattice. Such lattices can be viewed as minimal generating sets for von Neumann algebras. Using ideas from free probability theory, I will construct a large number of non-isomorphic Kadison-Singer lattices and give constructions to get new Kadison-Singer lattices from old. I will conclude the talk by giving structure results for Kadison-Singer algebras and lattices and indicating connections to problems in the theory of von Neumann algebras.