

İSTANBUL ANALYSIS SEMINARS

\mathcal{E}' AS AN ALGEBRA BY MULTIPLICATIVE CONVOLUTION AND HADAMARD-TYPE OPERATORS ON SMOOTH FUNCTIONS

Dietmar VOGT

Bergische Universität Wuppertal
Faculty of Mathematics and Natural Sciences

Abstract: We study the algebra $\mathcal{E}'(\mathbb{R}^d)$ equipped with the multiplication $(T \star S)(\varphi) = T_x(S_y(f(xy)))$ where $xy = (x_1y_1, \dots, x_dy_d)$. For open sets $\Omega', \Omega \subset \mathbb{R}^d$ we determine the distributions $T \in \mathcal{E}'(\mathbb{R}^d)$ such that $T \star \mathcal{E}'(\Omega') \subset \mathcal{E}'(\Omega)$, in particular, the T which operate on $\mathcal{E}'(\Omega)$. They are distributions whose support is contained in the dilation sets $V(\Omega', \Omega)$ or $V(\Omega)$, respectively. By transposition we obtain a characterization of the \star -convolution operators which send $C^\infty(\Omega)$ to $C^\infty(\Omega')$ and, in particular, the \star -convolution operators on $C^\infty(\Omega)$. These are called Hadamard-type operators and they are characterized by the property that they admit all monomials as eigenvectors. The algebra $M(\Omega)$ of such operators is a closed subalgebra of $L(C^\infty(\Omega))$ and we determine the topology induced from $L_b(C^\infty(\Omega))$ on $\mathcal{E}'(V(\Omega))$. We show that the algebra $M(\Omega)$ is isomorphic to an algebra of holomorphic functions around zero where multiplication is the classical Hadamard multiplication, that is, multiplication of the Taylor coefficients. Hadamard-type operators assigned to distributions with support $\{(1, \dots, 1)\}$ are called Euler operators and we study global solvability for such operators on open subsets of \mathbb{R}_+^d . Analogous problems for real analytic functions are studied in papers of Domański-Langenbruch and a paper of Domański-Langenbruch-Vogt. While the problems are analogous, the results, the methods and the difficulties to overcome are, in part, quite different.

Date: May 16, 2014

Time: 15:40

Place: Sabancı University, Karaköy Communication Center
Bankalar Caddesi 2, Karaköy 34420, İstanbul

İstanbul Analysis Seminars is supported by TÜBİTAK.