Harmonic Univalent Functions for Which Analytic Part is Convex Functions of Complex Order

Yaşar POLATOĞLU

Department of Mathematics and Computer Science,
İstanbul Kültür University, 34156 İstanbul, Turkey
e-mail: y.polatoglu@iku.edu.tr

Abstract

In a simply connected domain $\mathcal{U} \subset \mathbb{C}$ a complex-valued harmonic univalent function f has the representation $f(z) = h(z) + \overline{g(z)}$, where $h(z)$ and $g(z)$ are analytic function in $\mathbb{U}$ and called analytic part and co-analytic part of f, respectively.

Let $h(z) = a_0 + a_1 z + a_2 z^2 + \cdots$ and $g(z) = b_0 + b_1 z + b_2 z^2 + \cdots$ be analytic functions in the open unit disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$, if

$$|h'(z)|^2 - |g'(z)|^2 > 0 \quad \text{or} \quad |h'(z)|^2 - |g'(z)|^2 < 0,$$

then f is called sense-preserving harmonic function in $\mathbb{D}$. The class of sense-preserving harmonic functions in $\mathbb{D}$ with $a_0 = b_0 = 0$ and $a_1 = 1$ will be denoted by $\mathcal{S}_H$. Thus $\mathcal{S}_H$ contains standard class of univalent functions $\mathcal{S}$.

The aim of this paper is to investigate the subclass of $\mathcal{S}_H$, by choosing only these functions whose analytic parts are convex functions of complex order, for such mappings we will find distortion theorems.

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