Berger-Wang formula holds for collectively compact sets of linear operators

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Abstract. A family M in the space $B(X)$ of all linear bounded operators on an infinite dimensional Banach space X is called *collectively compact* if the set

$$M(U_X) = \{Tx : T \in M,\, x \in U_X\}$$

has compact closure in X, where U_X denotes the closed unit ball in X. Every precompact set of compact operators is collectively compact but the converse is not true in general [1]. We will show that the Berger-Wang formula $\rho(M) = r(M)$, where $\rho(M)$ and $r(M)$ are the joint spectral radius and the Berger-Wang spectral radius of M, respectively, holds for any collectively compact set M of operators, which was proven in [2] for precompact sets of compact operators. Furthermore, some invariant subspace results which are proven for precompact sets of compact operators in [2] will be generalized to collectively compact families of operators, using well-known techniques.

References

[1] P. M. Anselone, *Collectively compact operator approximation theory and applications to integral equations*, Prentice-Hall Inc., 1971.

[2] V. S. Shulman and Yu. V. Turovskii, "Joint spectral radius, operator semigroups and a problem of W. Wojtynski," *J. Funct. Anal.* **177** (2000), 383-441